Reduction formula for $\int \sin^n x\,dx$

Let $I_n = \int \sin^n x\,dx = \int \sin^{n-1}x \sin x\,dx$

$= \sin^{n-1}x(-\cos x) - \int -\cos x\,(n-1)\sin^{n-2}x \cos x\,dx$

$= -\cos x \cdot \sin^{n-1}x + (n-1)\int \sin^{n-2}x\,(1-\sin^2 x)\,dx$

$= -\cos x\,\sin^{n-1}x + (n-1)\int \sin^{n-2}x\,dx - (n-1)\int \sin^n x\,dx$

$\therefore I_n = -\cos x \cdot \sin^{n-1}x + (n-1)\int \sin^{n-2}x\,dx$
$- (n-1)\int \sin^n x\,dx.$

$\therefore I_n = -\cos x \sin^{n-1}x + (n-1) I_{n-2} - (n-1) I_n$

$\therefore n I_n = -\cos x \sin^{n-1}x + (n-1) I_{n-2}$

$I_n = -\frac{1}{n} \cdot \cos x \cdot \sin^{n-1}x + \frac{n-1}{n} I_{n-2}$

Reduction formula for $\int \tan^n x \, dx$

If $I_n = \int \tan^n x \, dx$

$$= \int \tan^{n-2} x \cdot \tan^2 x \, dx$$

$$= \int \tan^{n-2} x (\sec^2 x - 1) \, dx$$

$$= \int \tan^{n-2} x \sec^2 x - \int \tan^{n-3} x \, dx$$

$$= \int \tan^{n-2} x \, d(\tan x) - I_{n-2}$$

OR $I_n + I_{n-2} = \dfrac{\tan^{n-1} x}{n-1}$. Hence

$$(n-1)(I_n + I_{n-2}) = \tan^{n-1} x$$

Note Similarly the reduction formula for $\int \cot^m x \, dx$ can be obtained.